🏆 *reviewed paper*

Supporting Spatial Planning with Qualitative Configuration Analysis

Paolo Fogliaroni, Gerhard Navratil

(Dr. Paolo Fogliaroni, Vienna University or Technology, Department for Geodesy and Geoinformation, Gusshausstr. 27-29, A-1040 Vienna, Austria, paolo@geoinfo.tuwien.ac.at)

(Dr. Gerhard Navratil, Vienna University or Technology, Department for Geodesy and Geoinformation, Gusshausstr. 27-29, A-1040 Vienna, Austria, navratil@geoinfo.tuwien.ac.at)

1 ABSTRACT

Today, spatial planning strongly relies on computerized Spatial Information Systems (SIS) which, thank to continuous improvements, provides increasingly better techniques for handling, visualizing, and analyzing quantitative (geometric or geographic) spatial data. An important typology of instruments today largely lacking in SIS is concerned with the management and analysis of qualitative spatial information. Indeed, although quantitative spatial information is necessary when it comes to precise computations, a qualitative approach is to be preferred in situations where preciseness is not necessary or even undesired. This is typically the case when it comes to the interaction with human beings. In this paper we outline an extension for a generic SIS that enables the system to handle qualitative spatial information and discuss how this can support spatial planning in several ways.

2 INTRODUCION

Geographic Information Systems (GIS) and Computer-Aided Design (CAD) applications play a central role in spatial planning. Such computerized systems, indeed, provide increasingly sophisticated representational and analytical means which support spatial planners in carrying out improved land studies and in making more accurate decisions.

In spite of continuous improvements, however, GIS and CAD keep on relying mainly on quantitative spatial (i.e., geometric or geographic) representations, while some important features largely missing today are concerned with the management and the analysis of qualitative spatial information. Indeed, although quantitative analysis is necessary when it comes to precise computations, a qualitative approach is to be preferred in situations where preciseness is not necessary or even undesired. This is typically the case when it comes to the interaction with human beings, which, as argued in the past (K. Lynch, 1960), naturally represent and reason about space in a qualitative manner: for example, people prefer to express relative position between two objects resorting to predicates like "*left of*" and "*right of*" rather than reporting the angular distance among them. Predicates of this type are called *qualitative spatial relations*, and, accordingly, a set of them describing a spatial scene shall be called a qualitatively-described spatial configuration or, more simply, a *qualitative spatial configuration* (Fogliaroni, 2012).

In this paper we outline an extension for a generic Spatial Information System (SIS) that enables the system to handle qualitative spatial information and, more specifically, qualitative spatial configurations. We elaborate how this can benefit spatial planning in several ways:

(1) Natural planner-SIS interaction: the identification of a spatial plan location can be done either by describing in natural language or by sketching its position with respect to other elements of the embedding environment;

(2) Design phase support: according to the type of plan being designed, the extended SIS suggests a list of items that should be placed in the workspace as well as the best spatial arrangement;

(3) Public participation enabling: a spatial plan eventually yields a change in the structure of a real environment whose quality is best assessed by its users; assessment (at least partly) relies on the (qualitative) spatial configuration of environmental elements. Thanks to its capacity of qualitatively representing spatial configurations, the extended SIS allows for the collection of public feedback which can be used to enhance the support provided in the design phase.

The remainder of this paper is organized as follows. In Sections 3 and 4 we provide an overview on the relevant state of the art in Spatial Information Systems and Qualitative Spatial Representation and Reasoning, respectively. Section 5 reports about an extension for a SIS developed in (Fogliaroni, 2012) that enables the system to deal with qualitative spatial information. In Section 6 we further extend the system and discuss how it allows for supporting spatial planning whereas in Section 6.1 we present an approach for computing prototypical spatial plans out of previous designs. Section 7 is devoted to show how the described system can be integrated with standard web-based pooling platforms to refine plan prototypes according to public expectation. Finally, conclusions are drawn in Section 8.

3 SPATIAL INFORMATION SYSTEMS

With the term Spatial Information System (SIS) we shall intend, in the scope of this paper, any system of hardware and software elements that allows for surveying, storing, analyzing, manipulating, retrieving, sharing, and presenting spatial data. Two well-known examples are Geographic Information Systems (GIS) and Computer-Aided Design (CAD) applications. From a logical perspective a SIS consists of a series of specialized layers interacting and collaborating with each other; at the lowest level lays the storage layer: a support to persistently represent data in a computer. Data storing can be done by means of purposely designed file encodings—e.g., shapefiles (ESRI, 1998). Nonetheless, the usage of a spatial database in this role is the most common scenario, since its optimized access methods notably improve data storage, retrieval, and, thus, analysis performance.

A spatial database is a database furnished with data types and functions suited for handling spatial information, which, in the (geo)spatial domain is concerned with real and factitious entities: Real entities comprise physical objects, natural or artificial (e.g., a lake or a building), whereas factitious entities are those that are not physically distinguishable from surrounding ones or that refer to agglomerations of single entities as a unique concept (e.g., administrative districts or countries). An ontological perspective on this issue is taken by Smith (1995) who distinguishes between entities with bona fide and fiat boundaries, respectively. Modeling spatial information is done mainly following two approaches that in the literature (Longley et al., 2005; Worboys & Duckham, 2004) are typically referred to as field-based and object-based. The field-based approach looks at the properties that have to be modeled as continuous fields and discretizes them via the superimposition of a (typically regular) geometric structure (e.g., a grid). The resulting data model is commonly known as raster. Conversely, in the object-based approach the main focus is on the spatial entities, whose geometry is modeled by means of a series of line segments called vectors.

Any manipulation of spatial data (drawing, analysis, selection, and so forth) in a SIS corresponds, at the database level, to the employment of a series of so-called spatial operators: functions defined over a set of spatial objects that perform some kind of geometric operation over the input data and return a result. The OpenGIS Consortium released a set of specifications for spatial operators (OpenGIS Consortium, 1998) intended to serve as guidelines for any spatial database. An important typology of spatial operators is concerned with the determination of the topological relationship between pairs of objects. However, as pointed out in (Clementini & Di Felice 2000), topology is not the only spatial aspect one might be interested in. Thus, further operators should be defined to deal with other aspects of space (e.g., direction and distance).

4 QUALITATIVE SPATIAL REPRESENTATION AND REASONING

Qualitative Spatial Representation and Reasoning (QSR) (cf. Cohn & Renz, 2008 and Cohn & Hazarika 2001 for an overview) is a subfield of artificial intelligence that aims at developing spatial representation techniques and computational models capable of simulating human spatial cognition. Such computational models draw upon the development of so-called qualitative spatial calculi.

A qualitative spatial calculus is a sound mathematical structure providing (*i*) a finite set of symbols (called qualitative spatial relations) that can be used to model spatial scenes and (*ii*) a set of operations defined over such symbols that allow for performing symbolic reasoning. Typically, a qualitative calculus focuses on one specific aspect of space. More than thirty years of research efforts in the field of QSR led to the birth of a vast and heterogeneous set of theoretical frameworks addressing a variety of qualitative spatial aspects (e.g., topology, direction, and distance among the most fundamentals). One of the best-known qualitative spatial calculi is the 9-Intersection Model (9-IM) (Egenhofer, 1989) which defines the 8 topological relations that can hold over a pair of spatial regions homeomorphic to the closed unit disk. They are depicted in Fig. 1 and arranged according to their *conceptual neighborhood graph* (Egenhofer & Al-Taha, 1992).

The term conceptual neighborhood has been first introduced in (Freksa, 1992) and refers to the property of a qualitative relation holding among a sequence of objects to change into another relation when the spatial objects it relates are continuously deformed according to a certain type of transformation (e.g., a topological transformation: movement, stretching/shrinking, or twisting). For example, the conceptual neighborhood graph for 9-IM correctly shows that *Meet* is the nearest relation to *Disjoint* and *Overlap*. This means that, if at time t_i two objects are *Disjoint* and at time t_k they *Overlap*, there has to exist a time point t_j when they *Meet* such that $t_i<t_j<t_k$. In other words, the relational transition between *Disjoint* and *Overlap* goes through *Meet*.

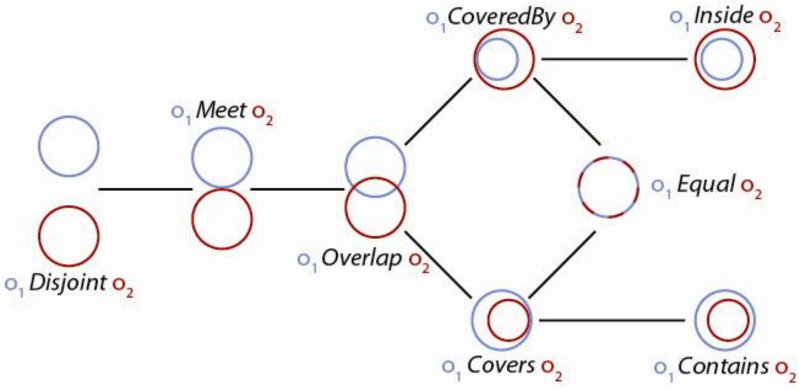

Fig. 1: the 8 possible topological relations between convex regions in the plane, as defined by the 9-Intersection Model (Egenhofer, 1989) and arranged to form the neighborhood graph.

As mentioned above, qualitative spatial calculi can be used to represent and reason about a configuration of spatial objects. One of the most spread methods resorts to the utilization of so-called Qualitative Constraint Networks (QCNs) (cf. Dechter, 2003 for a detailed discussion). A QCN can be seen as a labeled, directed (hyper)graph: each node represents a spatial object in the configuration; each (hyper)arc indicates which relation (reported in the label) holds among a sequence of spatial objects (those represented by the nodes connected by the hyperarc). For example, the spatial configuration depicted in Fig. 2(a) can be topologically described by the QCN in Fig. 2(c) taking relations (arc labels) from the 9-Intersection Model. Note that while the QCN associated to a geometrically described spatial configuration is unique, the opposite is not true: the same QCN can represent geometrically different spatial configurations. For example, the network in Fig. 2(c) also describes the configuration reported in Fig. 2(b), where object shapes, orientations, and relative positions are distorted and the only unchanged aspect is the topological one.

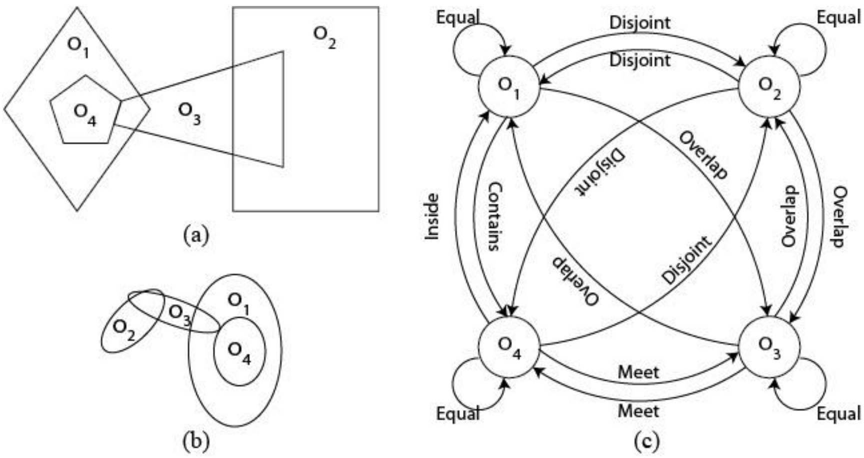

Fig. 2: The spatial configuration in (a) is uniquely represented by the qualitative (topological) constraint network in (c). However, note that the same network also represents the configuration in (b), meaning that the two configurations are topologically equivalent.

5 QUALITATIVE SPATIAL INFORMATION SYSTEM

In spite of continuous improvements in spatial data management and analysis, the interaction modalities between humans and Spatial Information Systems (SIS) keep remaining largely artificial and cognitively unnatural. One main motivation resides in the different ways SIS and humans deal with spatial information: SIS mainly draw upon quantitative spatial representations typically based on raster and vector data models, whereas human beings naturally resort to a qualitative and relational approach. For example, we use expressions like “the lake is to the right-hand side of the wood” or “is there a supermarket close to the university?” which qualitatively locate a spatial entity with respect to another.

Nowadays, such a gap in representation has to be plugged by the system user: He has to translate his mental, qualitative representation of space into a series of numerical constraints and to encode the latter in an artificial language which the system is capable of elaborating. As a result, this augments the cognitive effort of the interaction and, consequently, denies casual users (non-experts) the possibility to exploit the system.

One possible solution to this problem is presented in (Fogliaroni, 2012) where a theoretical and practical framework is detailed which allows for enabling any SIS relying on a spatial database to explicitly deal with qualitative spatial information: The database is enhanced with an extensible pool of qualitative spatial calculi in such a way that the spatial relations provided by the latter are available to the system as spatial operators. Such relational operators extend the artificial language of the system, allowing for a direct encoding of spatial descriptions naturally produced by a human. More specifically, they allow for modeling by means of Qualitative Constraint Networks (QCNs) both, a quantitatively described spatial dataset and a natural spatial description coming in either verbal (written or spoken utterances) or pictorial (sketch maps) format. The QCN representation of a spatial dataset is referred to as qualification (or more explicitly as qualitative dataset) whereas that of a natural spatial description is called Qualitative Spatial Relation Query (QSRQ).

QSRQs consist of sets of predicates of the form

(spatial relation, spatial object, …, spatial object)

taking spatial relations from those provided by the calculus pool; they are classified according to the level of indeterminacy of the spatial predicates they encode: the more numerous the elements of the predicate left unspecified, the harder the query to be solved. The hardest (realistic) query type consists of a series of predicates having only the spatial relation specified. A query of this category qualitatively describes the spatial arrangement of a set of undetermined objects (spatial variables) and, accordingly, it is called Qualitative Spatial Configuration Query (QSCQ). In (Fogliaroni, 2012) a set of algorithms and data structures has been designed that allows for efficiently solving such queries.

6 SUPPORTING SPATIAL PLANNING IN SPATIAL INFORMATION SYSTEMS

In this section we outline a further enhancement for the qualitative spatial information system (QSIS) discussed in Section 5 that allows for supporting spatial planners in the design phase. We suggest equipping the QSIS with two domain ontologies providing a hierarchical categorization of spatial objects and spatial plans, respectively. For the sake of exposition it is necessary to distinguish between the design of a spatial plan and its realization in the real world: we shall refer to the first as plan design and to the second as plan environment. Moreover, in the case the location for a spatial plan is not predetermined, but rather has to be decided by the planner, we also shall discriminate between workspace location and environment location: the first indicates the location a planner has to identify within a spatial dataset according to certain given constraints whereas the second denotes the actual location of a plan environment in the real world.

Spatial object categories define classes of spatial objects like dwelling, road, and grocery. Moreover, we assume that the ontology defines some categories like public green, industrial area, and residential area which describe agglomerates of spatial objects (factitious entities). Object categories can be used in place of generic spatial objects within a QSCQ to provide a more accurate definition of the searched configuration, although still general. For example, the following is a QSCQ describing the arrangement of two dwellings, one road, one grocery and one agglomeration of objects that is a public green zone:

{dwelling(o1) adjacent to road(o2), dwelling(o1) right of road(o2),

dwelling(o3) adjacent to road(o2),

grocery(o4) adjacent to road(o2), grocery(o4) close to dwelling(o1),

public green(o5) close to dwelling(o1), public green(o5) right of road(o2)}

Plan categories potentially coincide with object agglomeration categories and we shall assume that to each of them is associated a QSCQ describing the optimal spatial configuration for that specific plan category. In particular, such an optimal configuration defines the number of objects of each category that should be placed in the design (and, consequently, in the environment) as well as in which manner they should be spatially arranged.

A QSIS enhanced in this way can support the design of a spatial plan in several ways:

(1) In the case the environment location is not decided a priori and it is on the planner selecting an appropriate one, the planner can complement a standard search (based on object attributes like land usage or

land type) with a sketched or a verbally described spatial configuration (i.e., a QSCQ) that the workspace location has to satisfy. As an example let us consider the search for an optimal location of a residential area. Several conditions must be met: The noise level should be low, transport capacity of the nearby transportation network must be adequate, the impact on nature shall be limited, and there should be not excessive risk of flooding or avalanches. These conditions are expressed as a QSCQ and suitable areas can be found.

(2) Once the workspace location is decided and selected, the planner states the plan category he intends to design. The system exploits the optimal configuration associated to that plan category to suggest to the planner a list of spatial objects (divided by category) that should be placed in the workspace and how they should be arranged. Possibly, the system might already suggest an arrangement taking into consideration the morphology of the land parcels covered by the workplace. For example, a residential area comprises more than just buildings and gardens. A road network, green areas, public services, etc. are also necessary. Using basic predicates (e.g., green areas should be accessible for all inhabitants) a starting distribution can be computed.

(3) During the design phase, the planner can drag and drop new objects in the workspace or transform objects already placed. The system continuously parses the design and produces the corresponding QCN which is compared with the optimal configuration associated to the plan category. The outcome of the comparison is used to point out to the planner which elements does not fit the optimal configuration and for associating an overall “optimality” score to the current design. For example, one might design a road going through a park and, if the spatial arrangement of such elements would conflict with the one reported in the optimal configuration, the system would point this out to the planner. Of course is up to the planner deciding whether taking care of the suggestion (i.e., the arrangement was a mistake) or not (i.e., the arrangement was intended to be like that).

While the scenario described in point 2 does not require anything more than retrieving the optimal configuration associated with plan category and use it to produce a prototypical design, points 1 and 3 give raise to a number of more advanced issues. Particularly, in the scenario described in point 1, the system shall return a location perfectly matching the given constraints. However, if such a perfect matching does not exist a best matching location might be suggested to the planner. The problem raised in point 3 is concerned with similarity assessment.

Since qualitative constraint networks (QCNs) naturally lend themselves to be represented as hypergraphs, such problems can be stated as (sub)graph matching problems. In particular they are: perfect matching, best matching, and similarity measurement.

The problem of finding a perfect matching can be solved via an adaptation of famous Ullmann's subgraph isomorphism algorithm (Ullmann, 1976) as described in (Fogliaroni, 2012, Section 3.4.1). In (Wallgrun et al., 2010), another variant of Ullmann’s algorithm is used to find the best matching between a pair of QCNs. Although apparently very similar to the first problem, the problem tackled by Wallgrun et al. differs in two main points. (i) They look for best partial matchings rather than for complete matchings, namely, they try to match the highest number of (hyper)arcs. This means that they are faced with a maximum common subgraph problem instead of a subgraph isomorphism. (ii) They also consider ambiguous QCNs, i.e., hyperarcs may be labeled with disjunctions of relations. Accordingly, they employ an A* algorithm to drive the matching process, employing consistency checking as the main constituent of the forward checking function.

The last problem can be solved as a combination of the two techniques above in the following manner: If a perfect matching is found, the two networks are equivalent (i.e., maximum similarity score), otherwise the best partial matching is detected which has a number of unmatched hyperarcs. The qualitative relation encoded by each such hyperarc differs from the relation reported in the QCN describing the optimal configuration and the distance in the conceptual neighborhood graph between each such relation pair can be used to compute a similarity score between the two networks.

Let us resume the exemplary qualitative configuration given in Fig. 2 and refine it by including object categories as depicted in Fig. 3(c). The QSCQ reported in Fig. 3(a) perfectly matches such a configuration. Indeed, it is easy to verify that the only variable assignment that ensures the matching is the following {x1=o2, x2=o1, x3=o4}. Conversely, there is no way to assign all the variables of the QSCQ in Fig. 3(b) in such a way that all its arc labels match those in the given configuration. In this case only a partial assignment is possible that maximizes the number of matched variables, it is: {x2=o1, x3=o4}. Starting from this partial assignment a complete one can be generated. In the example it is: {x2=o1, x3=o4, x1=o2}. Such a complete assignment yields a mismatch between the arc (x1 Overlap x2) of the QSCQ and the arc (o2 Disjoint o1) of the configuration. The distance between the mismatching relations in the conceptual neighborhood graph (cf. Fig. 1) is equal to 2. Such a number can be used to assign a similarity score to the matching that is an index of how close the QSCQ is to the qualitative configuration (i.e., the optimality score).

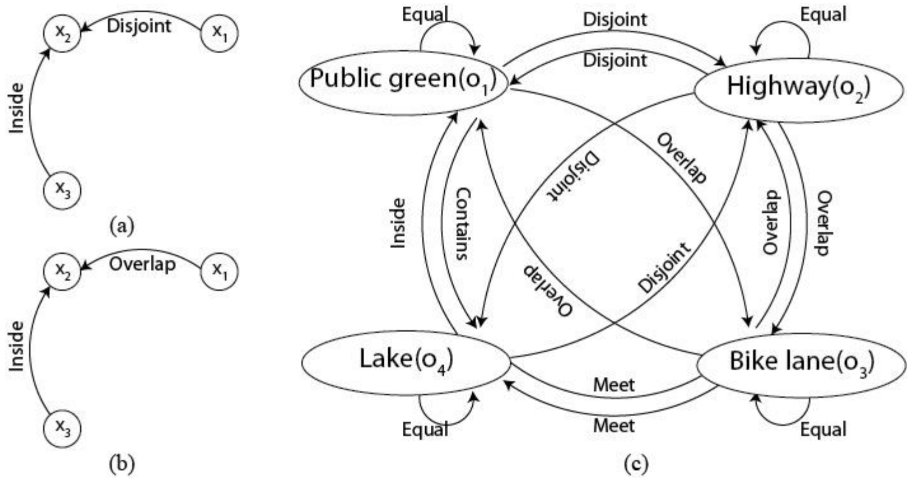

Fig. 3: The QSCQ in (a) perfectly matches the qualitative configuration in (c) while the QSCQ in (b) matches partially.

6.1 Computing the optimal plan

In this section we describe an approach to compute optimal configurations for spatial plan categories. Thanks to its features, the qualitative spatial information system we described in Section 6 is capable of maintaining a list of qualitative datasets for each plan category. Such datasets are, simply, the qualification of spatial plans designed within the system. As such, each of them shall embody at least a subset of common-sense, best-design, environmental, and urban rules that should be commonly fulfilled in the design phase.

To compute the optimal configuration for a given plan category we suggest to compare all the corresponding qualified datasets in the following manner.

(1) Normalize the qualitative datasets by

(1.1) computing the maximum common set of spatial objects (separated by category) involved in the plans;

(1.2) removing from each qualitative dataset the objects not contained in the common set as well as the relations in which they are involved.

(2) Compare the normalized datasets (via hypergraph matching) to find the minimum common hypergraph.

The resulting minimum common hypergraph represents the optimal spatial configuration we look for.

7 FITTING SPATIAL PLANS TO PUBLIC EXPECTATIONS

Public participatory online applications are web platforms developed to collect public opinions about a given topic of common interest. One recently established form of public participatory applications integrates standard pooling techniques with Geographic Information Systems to obtain public feedbacks about spatial matters. Such a form of public participation is referred to as Public Participatory Geographic Information System (PPGIS) and in (Poplin, 2012) it is shown how it can benefit spatial planning.

In the previous section we outline a method to compute for a given type of spatial plan an optimal configuration which conveys rules of good planning design. Now we show how our qualitative-enhanced spatial information system allows for automatically exploiting people feedback conveyed through a PPGIS application to let the optimal configuration fit public expectation.

The fact that human beings mentally map space in a qualitative manner means that the assessment of the quality of a certain environment is (at least partly) based on the presence (or absence) of a number of objects of a certain category and on their spatial arrangement within the environment. The quality assessment might also be influenced by a number of aspatial aspects including air pollution, price of a service, and weather conditions. However, in the scope of this work we shall disregard aspatial aspects and assume that the assessment of an environment can be directly associated to its qualitative representation (i.e., a QCN).

Accordingly, we suggest two slightly different approaches that resort to people feedbacks on the quality of an environment to compute optimal configurations.

The first approach draws on the idea that the quality of an environment (eventually resulting from a spatial plan) can be reliably assessed by its users: people working, living, or, more generally, carrying out an activity in the environment itself. Accordingly, we suggest asking users to evaluate the quality of a given environment by a numeric score. Such quality scores can be collected within the system and their average (or any other meaningful statistical operator) can be used to weight the plan design which the environment has been realized from. Every time the weight of a plan changes the optimal configuration for its category has to be recomputed as described in Section 6.1, this time taking into consideration the computed weights.

The second approach takes advantage of an online participatory application like, for example, the B3 project (http://www.geogameslab.de). B3 is a web platform that allows the registered user to actively participate in the planning process of Billstedt (a market place in Hamburg, Germany). The application consists of a 3D design environment displaying the actual (empty) location. The user can drag and drop environmental items from a predefined list or move them around within the scene until he obtains an environment satisfying his expectations. We suggest integrating our system to tools like B3 in such a way that the plan designed by each user can be qualified into a QCN which can be added to the list of qualitative datasets of a certain plan category. In such a way the optimal spatial configuration will fit not only good design rule, but also public expectation.

8 CONCLUSIONS

In this paper we presented an extension for a generic Spatial Information System (SIS) designed to benefit spatial planning. The extension builds on top of a framework presented in (Fogliaroni, 2012) which provides a standard SIS with qualitative spatial representation and reasoning capabilities. We suggested complementing such a framework by means of two domain ontologies providing hierarchical categorizations for spatial objects and spatial plans, respectively. The result is a qualitative-enhanced SIS that can benefit spatial planning design in several ways. As a basic functionality it provides more natural interaction means by bridging the representational gap existing among human beings (qualitative) and spatial information systems (quantitative). It also offers a mechanism to suggest the planner which environmental items should be placed in the design (according to the type of plan) and in which way they should be arranged in the workspace. The system is capable of assessing the plan being designed by comparing it with an optimal plan that accounts for good design rules; moreover it is also capable of pointing out which objects should be rearranged (if any). Finally we presented two approaches to integrate the suggested system with standard web-pooling techniques to collect public feedback and discussed how such a feedback can be used to refine the optimal plan; namely, to include public expectation in the spatial plan design phase.

9 REFERENCES

Clementini, E. & Di Felice, P. (2000). Spatial operators. SIGMOD Record, 29(3), 31–38.

Cohn, A.G., Hazarika, S.M. (2001). Qualitative spatial representation and reasoning: An overview. Fundamenta Informaticae 46(1-2), 1–29

Cohn, A.G., Renz, J. (2008). Qualitative spatial representation and reasoning. In: Frank van Harmelen, V.L., Porter, B. (eds.) Handbook of Knowledge Representation, Foundations of Artificial Intelligence, vol. 3, pp. 551–596. Elsevier

Dechter, R. (2003). Constraint processing. The Morgan Kaufmann Series in Artificial Intelligence, Morgan Kaufmann

Egenhofer, M.J. (1989). A formal definition of binary topological relationships. In: Litwin, W., Schek, H.J. (eds.) 3rd International Conference, FODO 1989 on Foundations of Data Organization and Algorithms, Lecture Notes in Computer Science, vol. 367, pp. 457–472. Springer-Verlag

Egenhofer, M. J. & Al-Taha, K. K. (1992). Reasoning About Gradual Changes of Topological Relationships. in: A. U. Frank, I. Campari, and U. Formentini (Eds.), Theories and Models of Spatio-Temporal Reasoning in Geographic Space, Pisa, Italy. Lecture Notes in Computer Science 639, pp. 196-219, Springer-Verlag, New York.

ESRI (1998). Shapefile technical description. Tech. rep., Environmental Systems Research Institute, Inc.

Fogliaroni, P. (2012). Qualitative Spatial Configuration Queries – Towards Next Generation Access Methods for GIS. Ph.D. Thesis (http://nbn-resolving.de/urn:nbn:de:gbv:46-00102731-11)

Freksa, C. (1992): Temporal reasoning based on semi-intervals. Artificial Intelligence 54, 199-227.

Lynch, K. (1960). The image of the city. MIT Press.

Longley, P., Goodchild, M., Maguire, D. & Rhind, D. (2005). Geographic Information Systems and Science. John Wiley & Sons, Ltd.

OpenGIS Consortium (1998). OpenGIS simple features specification for SQL. Tech. rep., OpenGIS Consortium (OGC).
Poplin, A. (2012). Playful public participation in urban planning: A case study for online serious games. Computers, Environment and Urban Systems.
Smith, B. (1995). On drawing lines on a map. Spatial Information Theory: A Theoretical Basis for GIS, 475–484.
Ullmann, J. (1976). An algorithm for subgraph isomorphism. Journal of the ACM (JACM), 23(1), 31–42.
Wallgrün, J.O., Wolter, D. & Richter, K.F. (2010). Qualitative matching of spatial information. In Proceedings of the 18th SIGSPATIAL International Conference on Advances in Geographic Information Systems, 300–309, ACM.
Worboys, M. & Duckham, M. (2004). GIS: A computing perspective (2nd edition). CRC Press, Inc.